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Dual-loop parity-time symmetric system with a rational loop length ratio

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By exploring the relationship between the gain/loss and the coupling coefficient, parity-time (PT) symmetry has been well explored in the photonics and optoelectronics fields to achieve unique functions, such as sidemode suppression, non-reciprocal light propagation, and unidirectional invisibility. In general, a PT-symmetric system has an architecture with two identical coupled resonators or loops. In this Letter, we explore the possibility of implementing a PT-symmetric system having an architecture with one resonator having a loop length that is a rational number of times the length of the other resonator, to increase the sidemode suppression ratio. A theoretical analysis is performed, which is validated by a proof-of-concept experiment in which a fiber ring laser having two loops with a length ratio being a rational number of 200/3, supporting single-longitudinal-mode lasing at 1555.88 nm, is demonstrated. Thanks to the non-identical loop lengths, the sidemode suppression ratio is increased, which is 53.2 dB in the experiment.

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Bender and Böttcher showed that in addition to Hamiltonians, non-Hermitian Hamiltonians could also exhibit real and positive spectra under the weaker condition of parity-time (PT) symmetry [1] in the quantum field. Thanks to the similar mathematical structure of the Schrödinger equation in the quantum area and Maxwell's theory of light under certain conditions, the concept of PT symmetry can be verified in the photonics field with ease. By exploiting the interactions between the gain, loss, and coupling strength of optical elements, novel optical effects have been found and utilized to realize various functions [2,3], including sidemode suppression in lasers and mode selection in optoelectronic oscillators (OEOs) [4–7], non-reciprocal light propagation in parallel waveguides and coupled microresonators [8,9], unidirectional invisibility in Bragg gratings and multilayer dielectrics [10,11], and coherent absorption in lasers [12,13].

PT symmetry has been implemented based on discrete devices and photonic integrated circuits (PICs). In general, an active resonator with a gain and a passive resonator with a loss having an identical physical geometry are coupled to form a PT-symmetric system. In the strong coupling region, the modes would split. These split modes will approach and finally coalesce when continuously decreasing the coupling strength or increasing the net gain/loss. For single-mode selection in a laser or an OEO, compared to the fundamental mode, the higher-order modes require extra gain/loss to reach the PT symmetry broken regime; thus, the fundamental mode will be the first to break PT symmetry and be amplified, while other modes will remain neutral. PT symmetry can also be employed to achieve non-reciprocal light propagation. Once PT symmetry is broken, one mode will experience a gain in the active resonator while the other will experience a loss in the passive resonator. Thus, the field is partially amplified in the active resonator no matter whether the input signal is injected from the active resonator or the passive one, and the amplified field will result in a gain-saturation nonlinearity, leading to non-reciprocal light propagation.

A PT symmetric system is usually implemented with two resonators or loops having an identical physical geometry to guarantee identical intrinsic resonant modes. It is challenging, especially when implementing using discrete components, to precisely match the resonator lengths. For a system with non-identical loop lengths, the intrinsic resonant modes in the two resonators will not match; thus, PT symmetry will not be achieved. However, if the loop lengths are not identical, but have a length ratio that is a rational number, some of the resonant modes will still be located at the same frequencies, and PT symmetry for these modes will still be achievable. In this Letter, we study the PT symmetry operation of an optical system having two coupled loops with the length ratio between the two loops being a rational number. We demonstrate that when the two loops have a length ratio that is a rational number, PT symmetry will be achieved. The main advantage of the structure is that the number of coupled modes is reduced significantly, making the sidemode suppression ratio (SMSR) significantly increased, enabling stable single-mode operation of the system. A proof-of-concept experiment is carried out. A dual-loop fiber ring laser having two coupled loops with the loop length ratio that is a rational number of 200/3 is implemented. When the PT symmetry is broken, single-longitudinal-mode lasing with a sub-kHz level linewidth at 1555.88 nm and an SMSR of 53.2 dB is realized.

For a dual-loop resonant system, the intrinsic resonant modes that are located at the same frequencies must satisfy the following phase-match condition [14]:

$$f_R = \frac{N}{\tau_1} = \frac{M}{\tau_2}, (N > M), \tag{1}$$

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where N and M are integers representing the Nth or Mth mode propagating in the long or short loops, respectively, and τ_1 and τ_2 are the round trip times of the long and the short loops, respectively. According to Eq. (1), the length of the long loop is $\frac{N}{M}$ times that of the short loop; that is, $\tau_1 = \frac{N}{M}\tau_2$, the effective mode spacing or free spectral range (FSR) is given by $\Delta f = N/[\gcd(N, M)\tau_1]$, where $\gcd(N, M)$ is the greatest common divisor of N and M. The effective FSR is increased and determined by the length of the short loop. The phenomenon is known as the Vernier effect.

However, a PT symmetry system is usually implemented by a pair of coupled resonators with equal lengths to ensure the same resonant frequencies, which can be mathematically modeled by coupled mode equations given by [15]

$$\frac{d}{dt}\begin{bmatrix} a_m \\ b_m \end{bmatrix} = \begin{bmatrix} -i\omega_m + \gamma_{a_m} & k\cos(\theta) \\ k\cos(\theta) & -i\omega_m + \gamma_{b_m} \end{bmatrix}\begin{bmatrix} a_m \\ b_m \end{bmatrix}, \quad (2)$$

where $\omega_m = 2\pi f_{R_m}$ is the angular frequency of the mth resonant mode in the dual loops, a_m and b_m are the amplitudes of the mth modes in the two loops, γ_{a_m} and γ_{b_m} represent gain and loss coefficients of the dual loops, θ is the angle between the linear polarization state of the two loops, and $k\cos(\theta)$ is the coupling strength between the two coupled loops.

If the lengths of the two resonators are not identical, and the loop length ratio becomes $\frac{N}{M}$, only some of the resonant modes will be located at the same frequencies. In this case, the resonant modes located at the same frequencies will be mutually coupled, then the coupled mode equations can be rewritten to be

$$\frac{d}{dt}\begin{bmatrix} a_p \\ b_q \end{bmatrix} = \begin{bmatrix} -i\omega_p + \gamma_{a_p} & k\cos(\theta) \\ k\cos(\theta) & -i\omega_q + \gamma_{b_q} \end{bmatrix}\begin{bmatrix} a_p \\ b_q \end{bmatrix}, \quad (3)$$

where p and q are integers representing the pth or qth modes propagating in the long or short loops, respectively. Since the pth mode in the long loop and the qth mode in the short loop are at the identical frequency $\omega = \omega_p = \omega_q = 2\pi p/\tau_1 = 2\pi q/\tau_2$, p and q should satisfy the relationship of $p = n \cdot N/\gcd(N, M)$, $q = n \cdot M/\gcd(N, M)$, where n is a positive integer. In such a system, once the gain of the pth mode in the long loop and loss of the qth mode in the long loop are tuned to be identical in magnitude, that is, $\gamma_{a_p} = -\gamma_{b_q} = \gamma$, the system is PT symmetric, and the eigenfrequencies of the system can be expressed by

$$\omega_c^{(1,2)} = \omega_c \pm \sqrt{k^2 - \gamma^2}. \quad (4)$$

If the length of the long loop is much longer than the short loop, that is, $\tau_1 \gg \tau_2$, multiple resonant modes in the long loop are within a resonant mode in the short loop, as shown in Fig. 1. In this case, the resonant mode in the short loop and those modes in the long loop are mutually coupled, and the coupling coefficients between the short-loop mode and the long-loop modes that are close to the short-loop mode can be regarded as a constant.

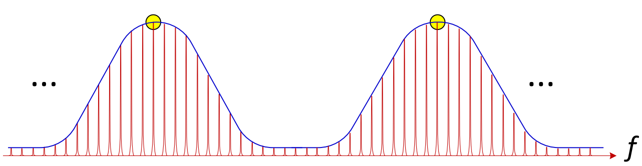

Fig. 1. Relationship between the resonant modes of the long (red) and the short (blue) loops.

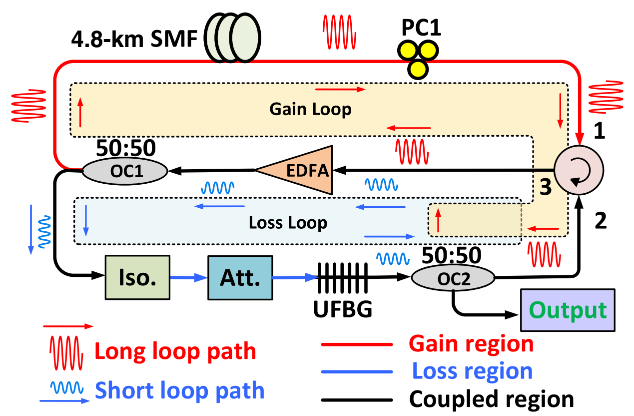

Fig. 2. Schematic diagram of a PT-symmetric fiber ring laser with non-identical loop lengths. SMF, single-mode fiber; PC, polarization controller; OC, optical coupler; EDFA, erbium-doped fiber amplifier; Iso., isolator; Att., attenuator; UFBG, uniform fiber Bragg grating.

However, due to the Vernier effect, the mode in the long loop that is best aligned with the mode in the short loop has the highest gain, and this pair of modes will first reach broken PT symmetry, and the effective FSR is $\Delta f = 2\pi/\tau_2$.

To verify our analysis, an optical system having two coupled loops with non-identical loop lengths, operating as a fiber ring laser, is shown in Fig. 2. As can be seen, the gain loop has an erbium-doped fiber amplifier (EDFA), a 4.8-km single-mode fiber (SMF), a polarization controller (PC1), an optical coupler (OC1), an optical circulator, and a uniform fiber Bragg grating (UFBG). The loss loop has an optical isolator, an optical attenuator, and the EDFA and the UFBG are shared by the loss loop. The output is taken from the loss loop through another OC (OC2). The mutual coupling is achieved at the UFBG where the transmitted light wave in the short loop is coupled with the partially reflected light wave in the long loop, and the coupling coefficient can be tuned by tuning PC1 to change the polarization state in the long fiber loop. In the meantime, the UFBG also works as an optical bandpass filter to determine the lasing wavelength.

It can be seen from Eq. (4), the eigenfrequencies will split when the coupling coefficient is larger than the gain/loss coefficient. However, once the gain/loss coefficient exceeds the coupling coefficient, the eigenfrequencies become a conjugate complex, corresponding to a pair of amplifying and decaying modes at ω_c. The amplifying mode will be enhanced, facilitating mode selection. Thus, single-longitudinal-mode lasing will be realized at ω_c.

Compared with a PT-symmetric laser having two loops with an identical loop length, the asymmetric dual loop design can effectively decrease the number of resonant modes, making mode selection much easier to achieve while maintaining a narrow linewidth at the lasing wavelength. In addition, the SMSR can also be increased due to the asymmetric dual loop design.

The operation of the asymmetric dual loop PT-symmetric fiber ring laser is experimentally studied based on the setup shown in Fig. 2. The key device to make mutual coupling between the two loops is the UFBG, which is designed to have a partial reflection. The transmission and reflection spectra of the UFBG measured using an optical vector network analyzer (OVNA) are shown in Fig. 3(a). As can be seen, the Bragg wavelength

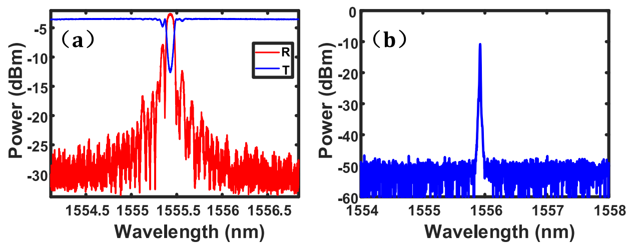

Fig. 3. (a) Transmission (red) and reflection (blue) spectra of the UFBG. (b) Measured optical spectrum of the generated light wave by using the UFBG in the dual-ring fiber ring laser.

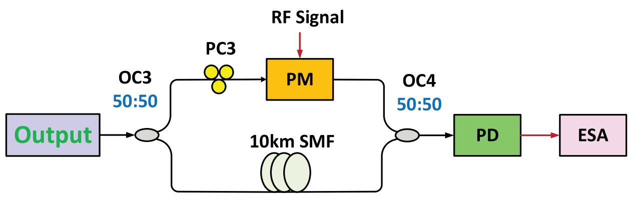

Fig. 4. Schematic diagram of the frequency-shifted self-heterodyne system. PM, phase modulator, PD, photodetector; ESA, electrical spectrum analyzer.

of the UFBG is 1555.4 nm, the bandwidth is 0.08 nm, and the reflection coefficient is 75%. By controlling the length ratio between the two loops to be 200/3, the effective FSR of the dual loops is 2.8 MHz. When the two loops are closed, and the gain and loss coefficients of the gain and loss loops are controlled balanced, PT symmetry is realized. Once the gain is greater than the coupling coefficient, the PT symmetry is broken, and single longitudinal mode lasing is achieved. Figure 3(b) illustrates the optical spectrum of the light generated by the dual-loop fiber ring laser, measured by an optical spectrum analyzer (OSA). As can be seen, a light emission at 1555.88 nm is generated. To evaluate the stability, the wavelength and power fluctuations are measured for a time duration of one hour. The results show that the fluctuations are within 3 pm and 0.12 dB.

Since the effective FSR is 2.8 MHz and the bandwidth of the UFBG is 0.08 nm, if no PT symmetry is involved in the mode selection process, the dual loop fiber ring laser will operate in multimode. The resolution of the OSA is too low to enable correct measurement of the single-mode operation of the fiber ring laser. To evaluate that the fiber ring laser is really operating in single mode enabled by the broken PT symmetry, we build a frequency-shifted self-heterodyne system, shown in Fig. 4, by which the single-mode operation of the fiber laser can be evaluated. The frequency-shifted self-heterodyne system consists of a phase modulator (PM), an SMF with a length of 10 km, two OCs (OC3 and OC4), and a photodetector (PD). A 1-GHz microwave signal is sent to the PM to shift the frequency of the beat signal between the two signals from two arms away from DC, ensuring an accurate measurement.

We first open the short loop by disconnecting the isolator in the short loop, then the system becomes a single-loop fiber ring laser with a long loop length. The loop length is 4.86 km and the FSR is 42 kHz. Since the FSR is much smaller than the bandwidth of the UFBG, multimode operation of the fiber ring laser will result. The RF beat signals with two different spans, measured by an electrical spectrum analyzer (ESA), are shown in Figs. 5(a)

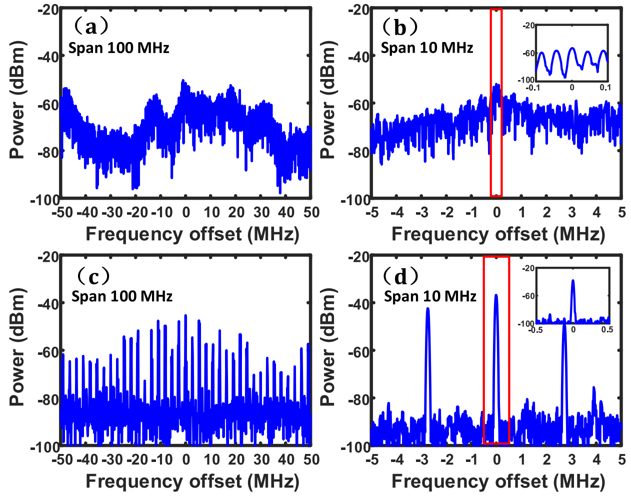

Fig. 5. Electrical spectra of the fiber ring laser with only the long loop closed measured at a span of (a)100 MHz and (b) 10 MHz. The electrical spectra of the fiber ring laser with only the short loop closed measured at a span of (c) 100 MHz and (d) 10 MHz.

and 5(b). It can be observed from the zoom-in view in the inset of Fig. 5(b) that the FSR of the long loop is approximately 42 kHz and no single-mode lasing is produced. Then, we open the long loop by disconnecting port 1 of the optical circulator to evaluate the operation of the fiber ring laser with only a short loop. The loop length is 73 m and the FSR is 2.8 MHz. Again, since the FSR is much smaller than the bandwidth of the UFBG, multimode operation of the fiber ring laser will still result. The RF beat signals with two different spans are shown in Figs. 5(c) and 5(d). Again, no single-mode lasing is produced. The results confirm that single-mode lasing is not possible even when the loop length is reduced to 73 m.

Then, the two loops are both closed and the gain in each loop is controlled to be higher than the loss. The system now becomes a fiber ring laser with two gain loops. Due to the Vernier effect, the effective FSR becomes 2.8 MHz, which is equal to that of the short loop. Figures 6(a) and 6(b) show the generated RF beat signals when both loops are closed. As can be seen, the fiber ring laser is operating in multimode. It is worth noting that in the zoom-in view of the fundamental mode in Fig. 6(b), two sidemodes with lower amplitudes are observed, which are the longitudinal modes of the long loop existing within the linewidth of the mode of the short loop. Compared with the number of resonant modes in the long loop, the number of resonant modes in the coupled loops is significantly decreased. However, the dual loop fiber ring laser is still operating in multimode because the effective FSR is much smaller than the bandwidth of the UFBG.

To realize single-longitudinal-mode lasing, PT symmetry should be introduced. By tuning the gain and loss coefficients of the dual loops to make them identical in magnitude, and the gain/loss coefficient to exceed the coupling coefficient, the system reaches the PT symmetry broken region. The corresponding electrical spectra are shown in Figs. 6(c) and 6(d). As can be seen, a microwave signal at 1 GHz is observed, and signals at other frequencies are significantly suppressed, which confirms that the fiber ring laser is operating in single longitudinal mode. The SMSR is measured to be 53.2 dB, which is very large thanks to the use of dual loops. In Fig. 6(d), the two sidemodes observed

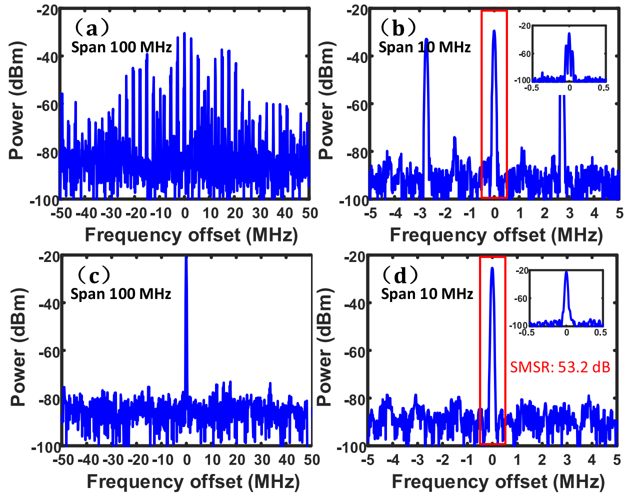

Fig. 6. Electrical spectra of the dual-loop fiber ring laser with each of the loops having a gain that is greater than the loss, measured at a span of (a) 100 MHz and (b) 10 MHz. The electrical spectra of the dual-loop fiber ring laser when the broken PT symmetry condition is satisfied, measured at a span of (c) 100 MHz and (d) 10 MHz.

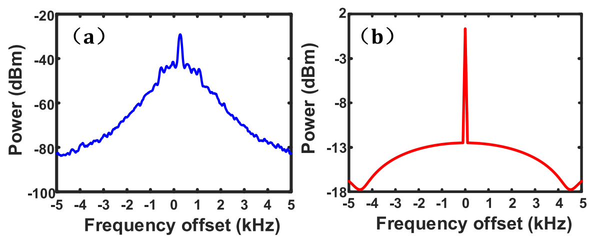

Fig. 7. (a) Electrical spectrum at the output of the frequency-shifted self-heterodyne system when the fiber ring laser is operating in single longitudinal mode. (b) Simulated power spectrum of the self-heterodyne signal.

in Fig. 6(b) caused by the long loop no longer exist, showing that PT symmetry can further increase the SMSR. Thanks to the long loop length, the linewidth of the light generated by the dual loop fiber ring laser when operating in single longitudinal mode is narrowed according to the Schawlow–Townes formula [16], which is confirmed by the experiment. Figure 7(a) shows the measured electrical spectrum. A simulated power spectrum of the self-heterodyne signal based on the analysis in Ref. [17] is also provided, as shown in Fig. 7(b). By comparing Figs. 7(a) and 7(b), we can conclude that the 3-dB linewidth of the proposed fiber ring laser is at the sub-kHz level. However, since a fiber used in the frequency-shifted self-heterodyne system has a length of 10 km, which is not long enough to fully destroy the light coherence, thus eliminating the interference between the two optical signals from the two arms, the linewidth measurement is not accurate. To obtain an accurate linewidth measurement, a fiber with a length of several thousand km should be used in the self-heterodyne system. A detailed discussion on the linewidth measurement based on a self-heterodyne system can be found in [17].

In conclusion, we have studied that a system having two coupled resonators with one resonator having a loop length that is a rational number of times the length of the other loop can have PT symmetry, which has been evaluated by a proof-of-concept experiment in which a fiber ring laser was implemented. The use of dual loops with non-identical lengths can take advantage of the principle of the Vernier effect, which can efficiently improve the SMSR. Furthermore, the effective FSR of the system is increased due to the Vernier effect, making the single-mode selection much easier while maintaining the narrow linewidth depending on the long-length loop. A dual-loop fiber ring laser having a rational length ratio of 200/3 was built up and tuned to be PT symmetry broken, and single-longitudinal-mode lasing at 1555.88 nm was established. A linewidth at a sub-kHz level and an SMSR of 53.2 dB were achieved. The concept of achieving PT symmetry with non-identical lengths offers a good starting point for constructing other PT symmetric systems to achieve functions such as non-reciprocal light propagation and unidirectional invisibility.

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Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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